chapter

3

The Standard Atmosphere

Sometimes gentle, sometimes capricious, sometimes awful, never the same for two moments together; almost human in its passions, almost spiritual in its tenderness, almost divine in its infinity.

John Ruskin, *The Sky*

Aerospace vehicles can be divided into two basic categories: atmospheric vehicles such as airplanes and helicopters, which always fly within the sensible atmosphere, and space vehicles such as satellites, the Apollo lunar vehicle, and deep-space probes, which operate outside the sensible atmosphere. However, space vehicles do encounter the earth's atmosphere during their blastoffs from the earth's surface and again during their reentries and recoveries after completion of their missions. If the vehicle is a planetary probe, then it may encounter the atmospheres of Venus, Mars, Jupiter, etc. Therefore, during the design and performance of any aerospace vehicle, the properties of the atmosphere must be taken into account.

The earth's atmosphere is a dynamically changing system, constantly in a state of flux. The pressure and temperature of the atmosphere depend on altitude, location on the globe (longitude and latitude), time of day, season, and even solar sunspot activity. To take all these variations into account when considering the design and performance of flight vehicles is impractical. Therefore, a *standard atmosphere* is defined in order to relate flight tests, wind tunnel results, and general airplane design and performance to a common reference. The standard atmosphere gives mean values of pressure, temperature, density, and other properties as functions of altitude; these values are obtained from experimental balloon and sounding-rocket measurements combined with a mathematical model of the atmosphere. To a reasonable degree, the standard atmosphere reflects average atmospheric conditions, but this is not its

main importance. Rather, its main function is to provide tables of common reference conditions that can be used in an organized fashion by aerospace engineers everywhere. The purpose of this chapter is to give you some feeling for what the standard atmosphere is all about and how it can be used for aerospace vehicle analyses.

We might pose the rather glib question: *Just what is the standard atmosphere?* A rather glib answer is: *The tables in Apps. A and B at the end of this book.* Take a look at these two appendixes. They tabulate the temperature, pressure, and density for different altitudes. Appendix A is in SI units, and App. B is in English engineering units. Where do these numbers come from? Were they simply pulled out of thin air by somebody in the distant past? Absolutely not. The numbers in these tables were obtained on a rational, scientific basis. One purpose of this chapter is to develop this rational basis. Another purpose is to show you how to use these tables.

The road map for this chapter is given in Fig. 3.1. We first run down the left side of the road map, establishing some definitions and an equation from basic physics (the hydrostatic equation) which are necessary tools for constructing the numbers in the standard atmosphere tables. Then we move to the right side of the road map and discuss how the numbers in the tables are actually obtained. We go through the construction of the standard atmosphere in detail. Finally, we define some terms that are derived from the numbers in the tables—the pressure, density, and temperature altitudes—which are in almost everyday use in aeronautics.

Finally, we note that the details of this chapter are focused on the determination of the standard atmosphere for earth. The tables in Apps. A and B are for the earth's atmosphere. However, the physical principles and techniques discussed in this chapter are also applicable to constructing model atmospheres for other planets, such as Venus, Mars, and Jupiter. So the applicability of this chapter reaches far beyond the earth.

It should be mentioned that several different standard atmospheres exist, compiled by different agencies at different times, each using slightly different experimental data in the models. For all practical purposes, the differences are insignificant below 30 km (100,000 ft), which is the domain of contemporary airplanes. A standard atmosphere in common use is the 1959 ARDC model atmosphere. (ARDC stands for the U.S. Air Force's previous Air Research and Development Command, which

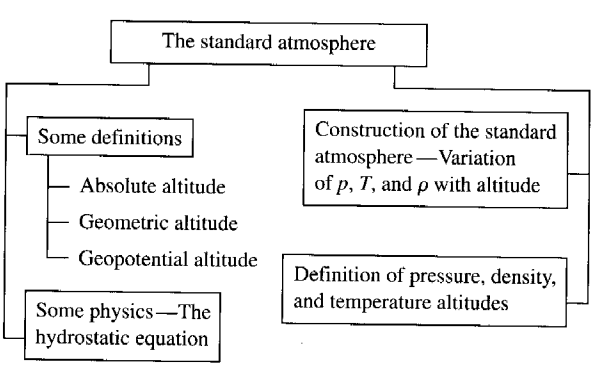

Figure 3.1 Road map for Chap. 3.

is now the Air Force Systems Command.) The atmospheric tables used in this book are taken from the 1959 ARDC model atmosphere.

3.1 DEFINITION OF ALTITUDE

Intuitively, we all know the meaning of altitude. We think of it as the distance above the ground. But like so many other general terms, it must be more precisely defined for quantitative use in engineering. In fact, in the following sections we define and use six different altitudes: absolute, geometric, geopotential, pressure, temperature, and density altitudes.

First, imagine that we are at Daytona Beach, Florida, where the ground is at sea level. If we could fly straight up in a helicopter and drop a tape measure to the ground, the measurement on the tape would be, by definition, the geometric altitude h_G, that is, the geometric height above sea level.

Now, if we bored a hole through the ground to the center of the earth and extended our tape measure until it hit the center, then the measurement on the tape would be, by definition, the *absolute altitude* h_a. If r is the radius of the earth, then $h_a = h_G + r$. This is illustrated in Fig. 3.2.

The absolute altitude is important, especially for space flight, because the local acceleration of gravity g varies with h_a. From Newton's law of gravitation, g varies inversely as the square of the distance from the center of the earth. By letting g_0 be

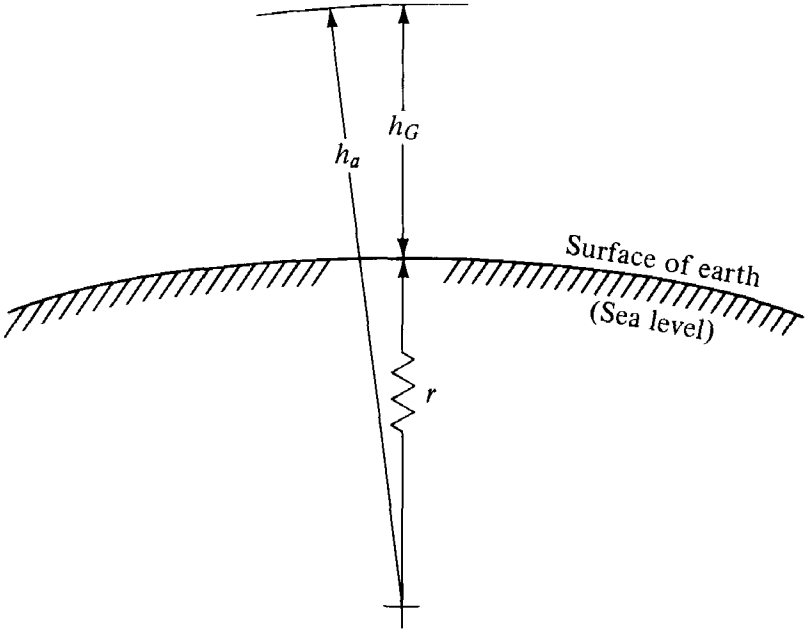

Figure 3.2 Definition of altitude.

the gravitational acceleration at *sea level*, the local gravitational acceleration g at a given absolute altitude h_a is

$$g = g_0 \left(\frac{r}{h_a}\right)^2 = g_0 \left(\frac{r}{r + h_G}\right)^2 \qquad [3.1]$$

The variation of g with altitude must be taken into account when you are dealing with mathematical models of the atmosphere, as discussed in the following sections.

3.2 HYDROSTATIC EQUATION

We will now begin to piece together a model which will allow us to calculate variations of p, ρ, and T as functions of altitude. The foundation of this model is the hydrostatic equation, which is nothing more than a force balance on an element of fluid at rest. Consider the small stationary fluid element of air shown in Fig. 3.3. We take for convenience an element with rectangular faces, where the top and bottom faces have sides of unit length and the side faces have an infinitesimally small height dh_G. On the bottom face, the pressure p is felt, which gives rise to an upward force of $p \times 1 \times 1$ exerted on the fluid element. The top face is slightly higher in altitude (by the distance dh_G), and because pressure varies with altitude, the pressure on the top face will be slightly different from that on the bottom face, differing by the infinitesimally small value dp. Hence, on the top face, the pressure $p + dp$ is felt. It gives rise to a

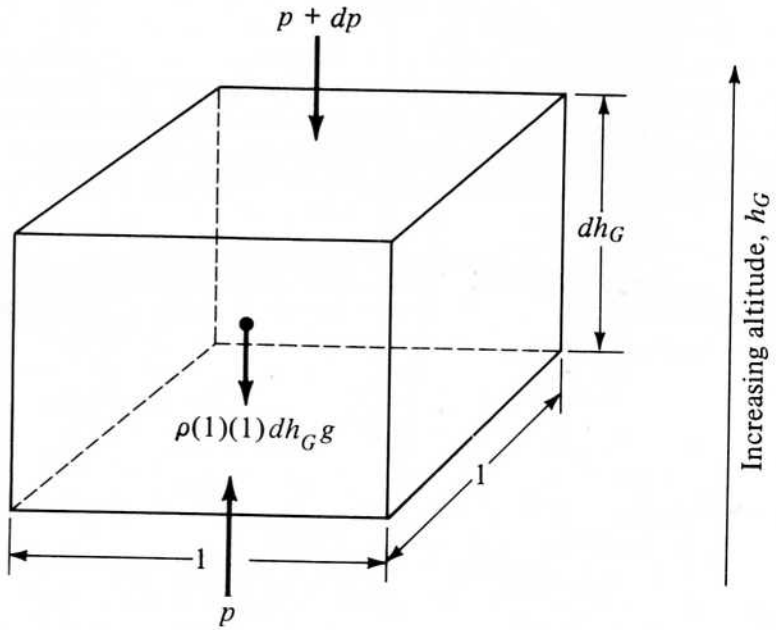

Figure 3.3 Force diagram for the hydrostatic equation.

downward force of $(p+dp)(1)(1)$ on the fluid element. Moreover, the volume of the fluid element is $(1)(1)\,dh_G = dh_G$, and since ρ is the mass per unit volume, then the mass of the fluid element is simply $\rho(1)(1)dh_G = \rho\,dh_G$. If the local acceleration of gravity is g, then the weight of the fluid element is $g\rho\,dh_G$, as shown in Fig. 3.3. The three forces shown in Fig. 3.3, pressure forces on the top and bottom, and the weight must balance because the fluid element is not moving. Hence

$$p = p + dp + \rho g\,dh_G$$

Thus

$$\boxed{dp = -\rho g\,dh_G} \qquad \textbf{[3.2]}$$

Equation (3.2) is the *hydrostatic equation* and applies to any fluid of density ρ, for example, water in the ocean as well as air in the atmosphere.

Strictly speaking, Eq. (3.2) is a differential equation; that is, it relates an infinitesimally small change in pressure dp to a corresponding infinitesimally small change in altitude dh_G, where in the language of differential calculus, dp and dh_G are differentials. Also note that g is a variable in Eq. (3.2); g depends on h_G as given by Eq. (3.1).

To be made useful, Eq. (3.2) should be integrated to give what we want, namely, the variation of pressure with altitude $p = p(h_G)$. To simplify the integration, we make the *assumption* that g is constant throughout the atmosphere, equal to its value at sea level g_0. This is something of a historical convention in aeronautics. At the altitudes encountered during the earlier development of human flight (less than 15 km or 50,000 ft), the variation of g is negligible. Hence, we can write Eq. (3.2) as

$$dp = -pg_0\,dh \qquad \textbf{[3.3]}$$

However, to make Eqs. (3.2) and (3.3) numerically identical, the altitude h in Eq. (3.3) must be slightly different from h_G in Eq. (3.2), to compensate for the fact that g is slightly different from g_0. Suddenly, we have defined a new altitude h, which is called the *geopotential altitude* and which differs from the geometric altitude. For the practical mind, geopotential altitude is a "fictitious" altitude, defined by Eq. (3.3) for ease of future calculations. However, many standard atmosphere tables quote their results in terms of geopotential altitude, and care must be taken to make the distinction. Again, geopotential altitude can be thought of as that fictitious altitude which is physically compatible with the assumption of $g = \text{const} = g_0$.

3.3 RELATION BETWEEN GEOPOTENTIAL AND GEOMETRIC ALTITUDES

We still seek the variation of p with geometric altitude $p = p(h_G)$. However, our calculations using Eq. (3.3) will give, instead, $p = p(h)$. Therefore, we need to relate h to h_G, as follows. Dividing Eq. (3.3) by (3.2), we obtain

$$1 = \frac{g_0}{g}\frac{dh}{dh_G}$$

or

$$dh = \frac{g}{g_0}dh_G \qquad \textbf{[3.4]}$$

We substitute Eq. (3.1) into (3.4):

$$dh = \frac{r^2}{(r+h_G)^2}dh_G \qquad \textbf{[3.5]}$$

By convention, we set both h and h_G equal to zero at sea level. Now, consider a given point in the atmosphere. This point is at a certain geometric altitude h_G, and associated with it is a certain value of h (different from h_G). Integrating Eq. (3.5) between sea level and the given point, we have

$$\int_0^h dh = \int_0^{h_G} \frac{r^2}{(r+h_G)^2}dh_G = r^2\int_0^{h_G}\frac{dh_G}{(r+h_G)^2}$$

$$h = r^2\left(\frac{-1}{r+h_G}\right)_0^{h_G} = r^2\left(\frac{-1}{r+h_G}+\frac{1}{r}\right) = r^2\left(\frac{-r+r+h_G}{(r+h_G)r}\right)$$

Thus

$$\boxed{h = \frac{r}{r+h_G}h_G} \qquad \textbf{[3.6]}$$

where h is geopotential altitude and h_G is geometric altitude. This is the desired relation between the two altitudes. When we obtain relations such as $p = p(h)$, we can use Eq. (3.6) to subsequently relate p to h_G.

A quick calculation using Eq. (3.6) shows that there is little difference between h and h_G for low altitudes. For such a case, $h_G \ll r$, $r/(r+h_G) \approx 1$, hence $h \approx h_G$. Putting in numbers, $r = 6.356766 \times 10^6$ m (at a latitude of 45°), and if $h_G = 7$ km (about 23,000 ft), then the corresponding value of h is, from Eq. (3.6), $h = 6.9923$ km, about 0.1 of 1 percent difference! Only at altitudes above 65 km (213,000 ft) does the difference exceed 1 percent. (Note that 65 km is an altitude at which aerodynamic heating of NASA's Space Shuttle becomes important during reentry into the earth's atmosphere from space.)

3.4 DEFINITION OF THE STANDARD ATMOSPHERE

We are now in a position to obtain p, T, and ρ as functions of h for the standard atmosphere. The keystone of the standard atmosphere is a *defined* variation of T with altitude, based on experimental evidence. This variation is shown in Fig. 3.4.

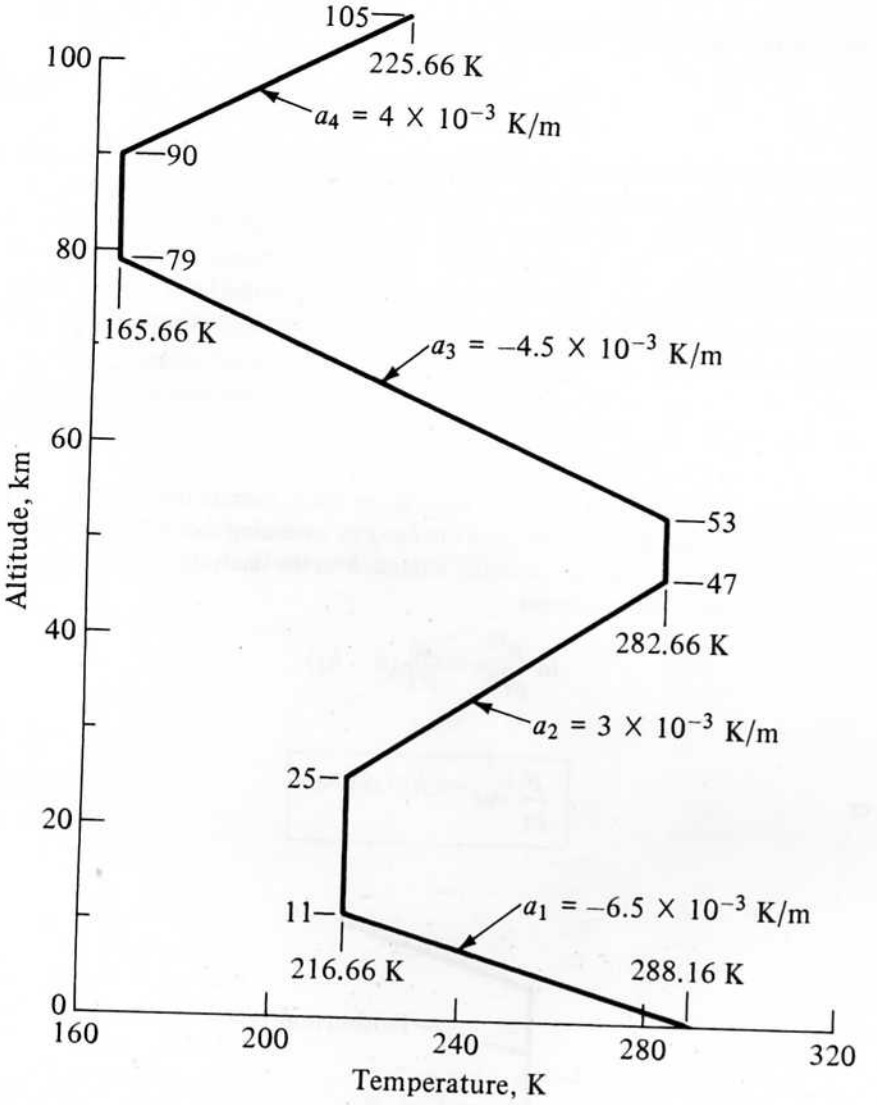

Figure 3.4 Temperature distribution in the standard atmosphere.

Note that it consists of a series of straight lines, some vertical (called the constant-temperature, or *isothermal*, regions) and others inclined (called the *gradient* regions). Given $T = T(h)$ as *defined* by Fig. 3.4, then $p = p(h)$ and $\rho = \rho(h)$ follow from the laws of physics, as shown below.

First, consider again Eq. (3.3):

$$dp = -\rho g_0 \, dh$$

Divide by the equation of state, Eq. (2.3):

$$\frac{dp}{p} = -\frac{\rho g_0\, dh}{\rho RT} = -\frac{g_0}{RT} dh \qquad \textbf{[3.7]}$$

Consider first the isothermal (constant-temperature) layers of the standard atmosphere, as given by the vertical lines in Fig. 3.4 and sketched in Fig. 3.5. The temperature, pressure, and density at the base of the isothermal layer shown in Fig. 3.5 are T_1, p_1, and ρ_1, respectively. The base is located at a given geopotential altitude h_1. Now consider a given point in the isothermal layer above the base, where the altitude is h. The pressure p at h can be obtained by integrating Eq. (3.7) between h_1 and h.

$$\int_{p_1}^{p} \frac{dp}{p} = -\frac{g_0}{RT} \int_{h_1}^{h} dh \qquad \textbf{[3.8]}$$

Note that g_0, R, and T are constants that can be taken outside the integral. (This clearly demonstrates the simplification obtained by assuming that $g = g_0 = \text{const}$, and therefore dealing with geopotential altitude h in the analysis.) Performing the integration in Eq. (3.8), we obtain

$$\ln \frac{p}{p_1} = -\frac{g_0}{RT}(h - h_1)$$

or

$$\boxed{\frac{p}{p_1} = e^{-[g_0/(RT)](h-h_1)}} \qquad \textbf{[3.9]}$$

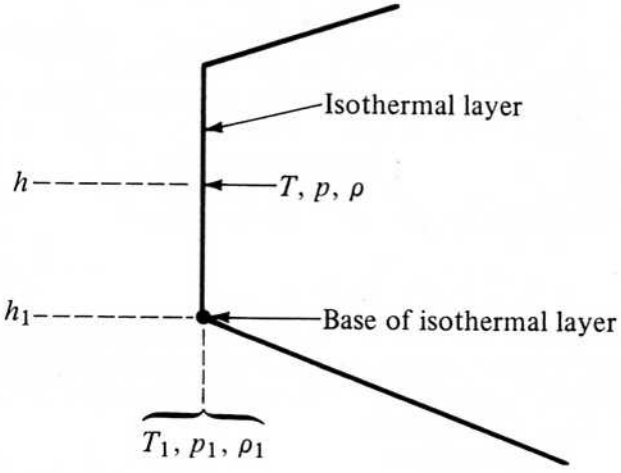

Figure 3.5 Isothermal layer.

From the equation of state

$$\frac{p}{p_1} = \frac{\rho T}{\rho_1 T_1} = \frac{\rho}{\rho_1}$$

Thus

$$\boxed{\frac{\rho}{\rho_1} = e^{-[g_0/(RT)](h-h_1)}} \qquad \textbf{[3.10]}$$

Equations (3.9) and (3.10) give the variation of p and ρ versus geopotential altitude for the isothermal layers of the standard atmosphere.

Considering the gradient layers, as sketched in Fig. 3.6, we find the temperature variation is linear and is geometrically given as

$$\frac{T - T_1}{h - h_1} = \frac{dT}{dh} \equiv a$$

where a is a *specified* constant for each layer obtained from the defined temperature variation in Fig. 3.4. The value of a is sometimes called the *lapse rate* for the gradient layers.

$$a \equiv \frac{dT}{dh}$$

Thus

$$dh = \frac{1}{a} dT$$

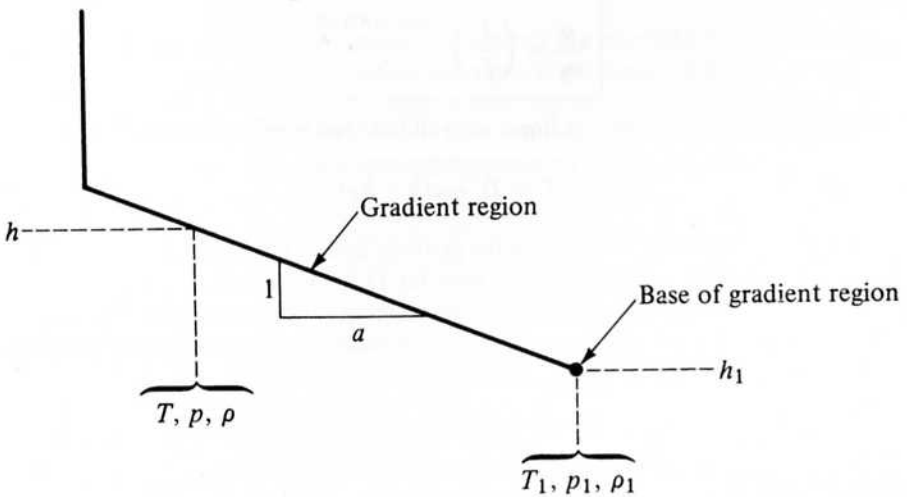

Figure 3.6 Gradient layer.

We substitute this result into Eq. (3.7):

$$\frac{dp}{p} = -\frac{g_0}{aR}\frac{dT}{T} \qquad \textbf{[3.11]}$$

Integrating between the base of the gradient layer (shown in Fig. 3.6) and some point at altitude h, also in the gradient layer, Eq. (3.11) yields

$$\int_{p_1}^{p}\frac{dp}{p} = -\frac{g_0}{aR}\int_{T_1}^{T}\frac{dT}{T}$$

$$\ln\frac{p}{p_1} = -\frac{g_0}{aR}\ln\frac{T}{T_1}$$

Thus

$$\boxed{\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-g_0/(aR)}} \qquad \textbf{[3.12]}$$

From the equation of state

$$\frac{p}{p_1} = \frac{\rho T}{\rho_1 T_1}$$

Hence, Eq. (3.12) becomes

$$\frac{\rho T}{\rho_1 T_1} = \left(\frac{T}{T_1}\right)^{-g_0/(aR)}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-[g_0/(aR)]-1}$$

or

$$\boxed{\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\{[g_0/(aR)]+1\}}} \qquad \textbf{[3.13]}$$

Recall that the variation of T is linear with altitude and is given the specified relation

$$\boxed{T = T_1 + a(h - h_1)} \qquad \textbf{[3.14]}$$

Equation (3.14) gives $T = T(h)$ for the gradient layers; when it is plugged into Eq. (3.12), we obtain $p = p(h)$; similarly from Eq. (3.13) we obtain $\rho = \rho(h)$.

Now we can see how the standard atmosphere is pieced together. Looking at Fig. 3.4, start at sea level ($h = 0$), where standard sea level values of pressure, density, and temperature—p_s, ρ_s, and T_s, respectively—are

$$p_s = 1.01325 \times 10^5 \text{ N/m}^2 = 2116.2 \text{ lb/ft}^2$$

$$\rho_s = 1.2250 \text{ kg/m}^3 = 0.002377 \text{ slug/ft}^3$$

$$T_s = 288.16 \text{ K} = 518.69^\circ\text{R}$$

These are the base values for the first gradient region. Use Eq. (3.14) to obtain values of T as a function of h until $T = 216.66$ K, which occurs at $h = 11.0$ km. With

DESIGN BOX

The first step in the design process of a new aircraft is the determination of a set of *specifications*, or *requirements*, for the new vehicle. These specifications may include such performance aspects as a stipulated maximum velocity at a given altitude, or a stipulated maximum rate of climb at a given altitude. These performance parameters depend on the aerodynamic characteristics of the vehicle, such as lift and drag. In turn, the lift and drag depend on the properties of the atmosphere. When the specifications dictate certain performance at a given altitude, this altitude is taken to be the standard altitude in the tables. Therefore, in the preliminary design of an airplane, the designer uses the standard atmosphere tables to define the pressure, temperature, and density at the given altitude. In this fashion, many calculations made during the preliminary design of an airplane contain information from the standard altitude tables.

these values of T, use Eqs. (3.12) and (3.13) to obtain the corresponding values of p and ρ in the first gradient layer. Next, starting at $h = 11.0$ km as the base of the first isothermal region (see Fig. 3.4), use Eqs. (3.9) and (3.10) to calculate values of p and ρ versus h, until $h = 25$ km, which is the base of the next gradient region. In this manner, with Fig. 3.4 and Eqs. (3.9), (3.10), and (3.12) to (3.14), a table of values for the standard atmosphere can be constructed.

Such a table is given in App. A for SI units and App. B for English engineering units. Look at these tables carefully and become familiar with them. They *are* the standard atmosphere. The first column gives the geometric altitude, and the second column gives the corresponding geopotential altitude obtained from Eq. (3.6). The third through fifth columns give the corresponding standard values of temperature, pressure, and density, respectively, for each altitude, obtained from the discussion above.

We emphasize again that the standard atmosphere is a reference atmosphere only and certainly does not predict the actual atmospheric properties that may exist at a given time and place. For example, App. A says that at an altitude (geometric) of 3 km, $p = 0.70121 \times 10^5$ N/m^2, $T = 268.67$ K, and $\rho = 0.90926$ kg/m^3. In reality, situated where you are, if you could right now levitate yourself to 3 km above sea level, you would most likely feel a p, T, and ρ different from the above values obtained from App. A. The standard atmosphere allows us only to reduce test data and calculations to a convenient, agreed-upon reference, as will be seen in subsequent sections of this book.

Example 3.1

Calculate the standard atmosphere values of T, p, and ρ at a geopotential altitude of 14 km.

Solution

Remember that T is a *defined* variation for the standard atmosphere. Hence, we can immediately refer to Fig. 3.4 and find that at $h = 14$ km,

$$T = 216.66 \text{ K}$$